Public Finance

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Outline of the class

Introduction

Lecture 2: Tax incidence

Lecture 3: Distortions and welfare losses

Lecture 4-6: Optimal labor income taxation

Intensive labor supply concepts

$$max_{c,z} u(\underset{+}{c}, \underset{-}{z}) \text{ subject to } c = z \cdot (1-\tau) + R$$

R is virtual income and τ marginal tax rate. FOC in $c, z \Rightarrow$

$(1-\tau)u_c + u_z = 0 \Rightarrow$ Marshallian labor supply $z = z(1-\tau, R)$

$$\text{Uncompensated elasticity} \quad \varepsilon^u = \frac{(1-\tau)}{z} \frac{\partial z}{\partial(1-\tau)}$$

$$\text{Income effects} \quad \eta = (1-\tau) \frac{\partial z}{\partial R} \leq 0$$

Intensive labor supply concepts

Substitution effects: Hicksian labor supply: $z^c(1-\tau, u)$ minimizes cost needed to reach u given slope $1-\tau \Rightarrow$

$$\text{Compensated elasticity} \quad \varepsilon^c = \frac{(1-\tau)}{z}\frac{\partial z^c}{\partial(1-\tau)} > 0$$

$$\text{Slutsky equation} \quad \frac{\partial z}{\partial(1-\tau)} = \frac{\partial z^c}{\partial(1-\tau)} + z\frac{\partial z}{\partial R} \Rightarrow \varepsilon^u = \varepsilon^c + \eta$$

Labor Supply Effects of Taxes and Transfers

Taxes and transfers change the slope $1 - T'(z)$ of the budget constraint and net disposable income $z - T(z)$ (relative to the no tax situation where $c = z$).

Positive MTR $T'(z) > 0$ reduces labor supply through substitution effects.

Net transfer ($T(z) < 0$) reduces labor supply through income effects.

Net tax ($T(z) > 0$) increases labor supply through income effects.

The perturbation method: Motivation

We seek to present an intuitive derivation of Diamond's ABC-formula that is sometimes referred to as "the perturbation method" or "tax reform". Complete version in Bierbrauer and Boyer (2018) and Bierbrauer, Boyer, Peichl (2021).

The overall logic of the perturbation method is explained in Saez (2001). We both show his formulation and an own formalization.

We think of individual's as facing an income tax schedule T, and as solving a consumer choice problem. A type ω-individual solves: Choose $y' \in \mathbb{R}_+$ so as to maximize $y' - T(y') - k(y', \omega)$.

Suppose that there is an initial tax schedule T_0 that individuals are facing. In the following, we provide a necessary condition for the optimality of T_0.

We consider a replacement of T_0 by a tax schedule T_1. T_1 has the same marginal tax rates as T_0, except for a small interval where marginal tax rates are increased by τ. Optimal behavior of an individuals is a function of ω, T_0 and τ.

The perturbation

More formally, T_1 is chosen in the following way: There exist cutoff levels of income y_a and y_b so that

i) For $y \leq y_a$, $T_1(y) = T_0(y)$.

ii) For $y \in (y_a, y_b)$, $T_1'(y) = T_0'(y) + \tau$. Hence, $T_1(y) = T_0(y) + \tau(y - y_a)$.

iii) For $y \geq y_b$, $T_1'(y) = T_0'(y)$. Hence, $T_1(y) = T_0(y) + \tau(y_b - y_a)$.

We think of τ as being close to zero, and of ω_b as being close to ω_a, i.e. we consider a small increase of the marginal tax rate for a small set of individuals.

In the following, we denote by $y^*(\omega, 0)$ the optimal behavior of a type ω-individual under the initial schedule T_0 and by $y^*(\omega, \tau)$ optimal behavior under the perturbed schedule.

A necessary condition for optimality I

The perturbation will lead to a change in tax revenue, given by

$$\Delta_R(\tau) := \int_{\underline{\omega}}^{\overline{\omega}} \Big\{ T_1(y^*(\omega,\tau)) - T_0(y^*(\omega,0)) \Big\} f(\omega) d\omega$$

We will redistribute this revenue in a lump sum fashion, i.e. any one individual's private goods consumption increases by $\Delta_R(\tau)$ which yields a welfare gain of

$$E[g(\omega)\Delta_R(\tau)] = \Delta_R(\tau)E[g(\omega)] = \Delta_R(\tau) \, .$$

(More generally, the revenue increase has to be weighted with the marginal cost of public funds, which equals 1 in a model with preferences that are quasilinear in consumption.)

A necessary condition for optimality II

This welfare gain has to be related to the welfare cost associated with moving from T_0 to T_1 and which is given by

$$\Delta_V(\tau) \quad = \quad \int_{\underline{\omega}}^{\overline{\omega}} g(\omega)\{V(\omega,\tau) - V(\omega,0)\}f(\omega)d\omega \ ,$$

where

$$V(\omega,\tau) := y^*(\omega,\tau) - T_1(y^*(\omega,\tau)) - k(y^*(\omega,\tau),\omega)$$

Thus, the total change in welfare associated with the perturbation is given

$$\Delta_R(\tau) + \Delta_V(\tau) \ .$$

A necessary condition for optimality III

A necessary condition

A necessary condition for the optimality of the initial tax schedule T_0 is that

$$\Delta'_R(0) + \Delta'_V(0) = 0 \, ,$$

so that, starting from T_0, an increase of marginal tax rates over some interval (y_a, y_b) does not increase welfare.

Analysis I

Let us denote by $\omega_a(\tau)$ and $\omega_a(0)$ the types who choose an income of y_a under the perturbed and the initial tax schedule, respectively.

More formally, $\omega_a(\tau')$, for $\tau' \in \{0, \tau\}$, is implicitly defined by the equation

$$1 - T_0'(y_a) - \tau' = k_1(y_a, \omega_a(\tau')) \ .$$

Analogously, we define $\omega_b(\tau')$, for $\tau' \in \{0, \tau\}$, by the equation

$$1 - T_0'(y_b) - \tau' = k_1(y_b, \omega_b(\tau')) \ .$$

Note that $\omega_a(\tau) > \omega_a(0)$ and $\omega_b(\tau) > \omega_b(0)$.

Analysis II

We can now decompose the set of types in the following way:

1. Individuals with types $\omega \leq \omega_a(0)$. We **assume** that their optimization problem – under both T_1 and T_0 – has an interior solution. Moreover, the solution is the same under both schedules, i.e. $y^*(\omega, 0) = y^*(\omega, \tau)$. Thus,

$$\Delta_R^1(\tau) := \int_{\underline{\omega}}^{\omega_a(0)} \{T_1(y^*(\omega,\tau)) - T_0(y^*(\omega,0))\} f(\omega) d\omega = 0 \, .$$

and

$$\Delta_V^1(\tau) := \int_{\underline{\omega}}^{\omega_a(0)} g(\omega)\{V(\omega,\tau) - V(\omega,0)\} f(\omega) d\omega = 0 \, .$$

Analysis III

2. Individuals with types in $(\omega_a(0), \omega_a(\tau))$. We **assume** that the optimization problem has an interior solution under T_0 and that these individuals choose $y^*(\omega, \tau) = y_a$ under the perturbed schedule. Thus,

$$\begin{aligned}\Delta_R^2(\tau) &:= \int_{\omega_a(0)}^{\omega_a(\tau)} \{T_1(y^*(\omega,\tau)) - T_0(y^*(\omega,0))\} f(\omega) d\omega \\ &= \int_{\omega_a(0)}^{\omega_a(\tau)} \{T_0(y_a) - T_0(y^*(\omega,0))\} f(\omega) d\omega \,.\end{aligned}$$

and

$$\begin{aligned}\Delta_V^2(\tau) &:= \int_{\omega_a(0)}^{\omega_a(\tau)} g(\omega)\{V(\omega,\tau) - V(\omega,0)\} f(\omega) d\omega \\ &= \int_{\omega_a(0)}^{\omega_a(\tau)} g(\omega)\{y_a - T_0(y_a) - k(y_a,\omega) - V(\omega,0)\} f(\omega) d\omega \,.\end{aligned}$$

Analysis IV

3. Individuals with types in $[\omega_a(\tau), \omega_b(\tau)]$. We **assume** that their optimization problem – under both T_1 and T_0 – has an interior solution, and define

$$\begin{aligned}\Delta_R^3(\tau) &:= \int_{\omega_a(\tau)}^{\omega_b(\tau)} \{T_1(y^*(\omega,\tau)) - T_0(y^*(\omega,0))\} f(\omega) d\omega \\ &= \int_{\omega_a(\tau)}^{\omega_b(\tau)} \{T_0(y^*(\omega,\tau)) - T_0(y^*(\omega,0)) + \tau(y^*(\omega,\tau) - y_a)\} f(\omega) d\omega \,.\end{aligned}$$

and

$$\Delta_V^3(\tau) := \int_{\omega_a(\tau)}^{\omega_b(\tau)} g(\omega)\{V(\omega,\tau) - V(\omega,0)\} f(\omega) d\omega \,.$$

Analysis V

4. Individuals with types $\omega \geq \omega_b(\tau)$. We **assume** that their optimization problem – under both T_1 and T_0 – has an interior solution. Moreover, the solution is the same under both schedules, i.e. $y^*(\omega, 0) = y^*(\omega, \tau)$. Thus,

$$\begin{aligned}\Delta_R^4(\tau) &:= \int_{\omega_b(\tau)}^{\overline{\omega}} \{T_1(y^*(\omega,\tau)) - T_0(y^*(\omega,0))\} f(\omega) d\omega \\ &= (1 - F(\omega_b(\tau)))\tau(y_b - y_a) \ .\end{aligned}$$

and

$$\begin{aligned}\Delta_V^4(\tau) &:= \int_{\omega_b(\tau)}^{\overline{\omega}} g(\omega)\{V(\omega,\tau) - V(\omega,0)\} f(\omega) d\omega \\ &= \int_{\omega_b(\tau)}^{\overline{\omega}} g(\omega)\{-T_1(y^*(\omega,\tau)) + T_0(y^*(\omega,0))\} f(\omega) d\omega \ . \\ &= -(1 - F(\omega_b(\tau)))G(\omega_b(\tau))\tau(y_b - y_a) \ .\end{aligned}$$

Analysis VI

Note that

$$\Delta_R'(\tau) = \Delta_R^{1\prime}(\tau) + \Delta_R^{2\prime}(\tau) + \Delta_R^{3\prime}(\tau) + \Delta_R^{4\prime}(\tau)$$

and

$$\Delta_V'(\tau) = \Delta_V^{1\prime}(\tau) + \Delta_V^{2\prime}(\tau) + \Delta_V^{3\prime}(\tau) + \Delta_V^{4\prime}(\tau)\,.$$

Analysis VII

Observation 1

1. $\Delta_R^{1\,\prime}(0) = \Delta_V^{1\,\prime}(0) = 0.$
2. $\Delta_R^{2\,\prime}(0) = \Delta_V^{2\,\prime}(0) = 0.$
3. $\Delta_R^{3\,\prime}(0) = \int_{\omega_a(0)}^{\omega_b(0)} \{T_0'(y^*(\omega,0))y_\tau^*(\omega,0) + y^*(\omega,0) - y_a\} f(\omega) d\omega$ and
   $\Delta_V^{3\,\prime}(0) = -\int_{\omega_a(0)}^{\omega_b(0)} g(\omega)\{y^*(\omega,0) - y_a\} f(\omega) d\omega.$
4. $\Delta_R^{4\,\prime}(0) = (1 - F(\omega_b(0)))(y_b - y_a)$ and
   $\Delta_V^{4\,\prime}(0) = -(1 - F(\omega_b(0)))G(\omega_b(0))(y_b - y_a).$

Analysis VIII

Upon collecting terms, we find that

$$\Delta_R^{3\,\prime}(0) + \Delta_V^{3\,\prime}(0) = \int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(\omega,0)) y_\tau^*(\omega,0) f(\omega) d\omega + \int_{\omega_a(0)}^{\omega_b(0)} (1-g(\omega))\{y^*(\omega,0) - y_a\} f(\omega) d\omega$$

and

$$\Delta_R^{4\,\prime}(0) + \Delta_V^{4\,\prime}(0) = (1 - F(\omega_b(0)))(1 - G(\omega_b(0)))(y_b - y_a)\,. \tag{1}$$

Thus,

$$\Delta_R{}'(0) + \Delta_V{}'(0) = \int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(\omega,0)) y_\tau^*(\omega,0) f(\omega) d\omega + \int_{\omega_a(0)}^{\omega_b(0)} (1-g(\omega))\{y^*(\omega,0) - y_a\} f(\omega) d\omega + (1 - F(\omega_b(0)))(1 - G(\omega_b(0)))(y_b - y_a)\,.$$

Analysis IX

Observation 2

For any pair y_a and y_b with $y_b > y_a$, an optimal tax system needs to satisfy the following condition

$$\begin{aligned} 0 &= \int_{\omega_a(0)}^{\omega_b(0)} T_0'(y^*(\omega,0))y_\tau^*(\omega,0)f(\omega)d\omega \\ &+ \int_{\omega_a(0)}^{\omega_b(0)} (1-g(\omega))\{y^*(\omega,0)-y_a\}f(\omega)d\omega \\ &+ (1-F(\omega_b(0)))(1-G(\omega_b(0)))(y_b-y_a)\ . \end{aligned} \tag{2}$$

Note that this is a property of the unperturbed tax system T_0. We suppress the emphasis that expressions are evaluated for $\tau = 0$ in the following. This will simplify our notation and not lead to confusion.

Analysis X

Since $y_b = y^*(\omega_b)$ and $y_a = y^*(\omega_a)$, we can reformulate Observation 2 as follows:

Observation 3

For any pair ω_a and ω_b with $\omega_b > \omega_a$, an optimal tax system needs to satisfy the following condition

$$
\begin{aligned}
0 \quad = \quad & \int_{\omega_a}^{\omega_b} T_0'(y^*(\omega)) y_\tau^*(\omega) f(\omega) d\omega \\
& + \int_{\omega_a}^{\omega_b} (1 - g(\omega)) \{y^*(\omega) - y^*(\omega_a)\} f(\omega) d\omega \\
& + (1 - F(\omega_b))(1 - G(\omega_b))(y^*(\omega_b) - y^*(\omega_a)) \, .
\end{aligned} \tag{3}
$$

We can differentiate equation (3) with respect to ω_a: Since, for given ω_b, (3) has to hold for all ω_a, the value of the right-hand side of (3) must not change, if we change ω_a slightly. This yields

Analysis XI

Observation 4

For all $\omega_a < \omega_b$ it has to hold that

$$\begin{aligned} 0 &= -T_0'(y^*(\omega_a))y^*_\tau(\omega_a)f(\omega_a) \\ &\quad - \int_{\omega_a}^{\omega_b}(1-g(\omega))y^*_\omega(\omega_a)f(\omega)d\omega \\ &\quad -(1-F(\omega_b))(1-G(\omega_b))y^*_\omega(\omega_a)\;. \end{aligned} \tag{4}$$

Analysis XII

We are particularly interested in the limit that is obtained as ω_a converges to ω_b: If y^* is a continuous function and y^*_ω is bounded, this yields

$$0 = -T_0'(y^*(\omega_b))y^*_\tau(\omega_b)f(\omega_b) - (1-F(\omega_b))(1-G(\omega_b))y^*_\omega(\omega_b)\,. \tag{5}$$

or

$$T_0'(y^*(\omega_b)) = -\frac{1-F(\omega_b)}{f(\omega_b)}(1-G(\omega_b))\frac{y^*_\omega(\omega_b)}{y^*_\tau(\omega_b)}\,. \tag{6}$$

or, upon exploiting the first-order condition $1-T_0'(y^*(\omega_b)) = k_1(y^*(\omega_b),\omega_b)$,

$$\frac{T_0'(y^*(\omega_b))}{1-T_0'(y^*(\omega_b))} = -\frac{1-F(\omega_b)}{f(\omega_b)}(1-G(\omega_b))\frac{1}{k_1(y^*(\omega_b),\omega_b)}\frac{y^*_\omega(\omega_b)}{y^*_\tau(\omega_b)}\,. \tag{7}$$

Analysis XIII

We derive expressions for $y^*_\omega(\omega_b)$, and $y^*_\tau(\omega_b)$: Remember that $y^*(\omega,\tau)$ is implicitly defined by the equation

$$1 - T'(y^*(\omega,\tau)) - \tau = k_1(y^*(\omega,\tau),\omega)\ .$$

Differentiating this equation with respect to τ and ω, respectively, yields

$$y^*_\tau(\omega,\tau) = -\left(T''(y^*(\omega,\tau)) + k_{11}(y^*(\omega,\tau),\omega)\right)^{-1}$$

and

$$y^*_\omega(\omega,\tau) = -k_{12}(y^*(\omega,\tau),\omega)\left(T''(y^*(\omega,\tau)) + k_{11}(y^*(\omega,\tau),\omega)\right)^{-1}\ .$$

Hence,

$$\frac{y^*_\omega(\omega,\tau)}{y^*_\tau(\omega,\tau)} = k_{12}(y^*(\omega,\tau),\omega)\ ,$$

and, therefore,

$$\frac{y^*_\omega(\omega_b)}{y^*_\tau(\omega_b)} := \frac{y^*_\omega(\omega_b,0)}{y^*_\tau(\omega_b,0)} = k_{12}(y^*(\omega_b,0),\omega_b) = k_{12}(y^*(\omega_b),\omega_b)\ . \tag{8}$$

Analysis XIV

Upon substituting this into (7) we obtain:

$$\frac{T_0'(y^*(\omega_b))}{1-T_0'(y^*(\omega_b))} \quad = \quad -\frac{1-F(\omega_b)}{f(\omega_b)}(1-G(\omega_b))\frac{k_{12}(y^*(\omega_b),\omega_b)}{k_1(y^*(\omega_b),\omega_b)}\ . \tag{9}$$

The following Proposition summarizes our results:

Proposition 7

Suppose that T_0 is an optimal tax schedule and that it generates a continuous income function y^*. Then for any $\omega \in \Omega$ it has to be the case that

$$\frac{T_0'(y^*(\omega))}{1-T_0'(y^*(\omega))} \quad = \quad -\frac{1-F(\omega)}{f(\omega)}(1-G(\omega))\frac{k_{12}(y^*(\omega),\omega)}{k_1(y^*(\omega),\omega)}\ . \tag{10}$$

Note that this is the same as Proposition 5.

The perturbation method - Sufficient Statistics Approach

We will now show the perturbation method as explained in Saez (2001).

Excellent treatments of this approach:

Pedagogic in Piketty and Saez, (2013, Handbook); Lehmann (2013) Revue française d'Économie (intensive and extensive); Chetty (2009).

Very general treatment in Golosov, Tsyvinski and Werquin (2014).

The perturbation method - Optimal Non-Linear Income Tax

- Consider general problem of setting optimal $T(z)$

(1) Lump sum grant given to everybody equal to $-T(0)$

(2) Marginal tax rate schedule $T'(z)$ describing how (a) lump-sum grant is taxed away, (b) how tax liability increases with income

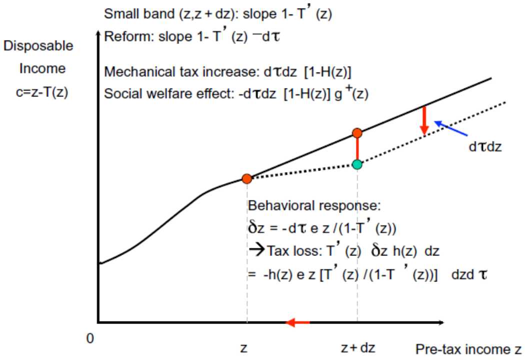
Assume away income effects $\varepsilon^c = \varepsilon^u = e$

- Let $H(z)$ = CDF of income [population normalized to 1] and $h(z)$ its density [endogenous to $T(.)$]

- Let $g(z)$ = social marginal value of consumption for taxpayers with income z in terms of public funds [formally $g(z) = G'(u) \cdot u_c / \lambda$]: no income effects $\Rightarrow \int g(z)h(z)dz = 1$ since giving \$1 to all costs \$1 (population has measure 1) and increase SWF (in \$ terms) by $\int g(z)h(z)dz$

  Redistribution valued $\Rightarrow g(z)$ decreases with z

- Let $G(z)$ be the **average** social marginal value of consumption for taxpayers with income above z [$G(z) = \int_z^\infty g(s)h(s)ds / (1 - H(z))$]


Small band (z,z + dz): slope 1- T' (z)
Reform: slope 1- T' (z) − dτ
Disposable
Income
c=z-T(z)
Mechanical tax increase: dτdz [1-H(z)]
Social welfare effect: -dτdz [1-H(z)] g +(z)
dτdz
Behavioral response:
δz = -dτ e z /(1-T' (z))
→Tax loss: T' (z) δz h(z) dz
= -h(z) e z [T' (z) /(1-T ' (z))] dzd τ
0
z
z+ dz
Pre-tax income z

- Consider small reform: increase T' by $d\tau$ in small band $(z, z + dz)$

- Mechanical revenue effect

$$dM = dzd\tau(1 - H(z))$$

- Mechanical welfare effect

$$dW = -dzd\tau(1 - H(z))G(z)$$

- Behavioral effect: substitution effect δz inside small band $[z, z + dz]$:

$$dB = h(z)dz \cdot T' \cdot \delta z = -h(z)dz \cdot T' \cdot d\tau \cdot \varepsilon(z) \cdot z/(1 - T')$$

- Optimum $dM + dW + dB = 0$

- Optimal tax schedule satisfies:

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon(z)} \left(\frac{1-H(z)}{zh(z)}\right) [1-G(z)]$$

- $T'(z)$ decreasing in $g(z')$ for $z' > z$ [redistributive tastes]
- $T'(z)$ decreasing in $\varepsilon_{(z)}$ [efficiency]
- $T'(z)$ decreasing in $h(z)/(1-H(z))$ [density]

Negative Marginal Tax Rates Never Optimal

- Suppose $T' < 0$ in band $[z, z + dz]$
- Increase T' by $d\tau > 0$ in band $[z, z + dz]$
- $dM + dW > 0$ because $G(z) < 1$ for any $z > 0$
  - Without income effects, $G(0) = 1$
  - Value of lump sum grant to all equals value of public good
  - Concave SWF: $G'(z) < 0$
- $dB > 0$ because $T'(z) < 0$ [smaller efficiency cost]
- Therefore $T'(z) < 0$ cannot be optimal
  - Marginal subsidies also distort local incentives to work
  - Better to redistribute using lump sum grant

Notes for further reading

Often Pontryagin's Maximium Principle/ Theory of optimal control is applied to obtain a characterization of optimal income taxes. See, for instance, Hellwig (2007), the textbook by or Salanié. These treatments are also more general in that specific assumptions on preferences such as quasi-linearity or additive separability are avoided.

You will see this method in Part II next week.

Income taxation and labor supply decisions

- Labor supply elasticity is a parameter of fundamental importance for income tax policy:

  Optimal tax rate depends inversely on the compensated wage elasticity of labor supply.

- Surveys in labor economics: Blundell and MaCurdy (1999) Handbook of Labor Economics.
- Surveys in public economics: Moffitt (2003) Handbook of Public Economics, Saez, Slemrod, and Giertz (2011).

- Micro and Macro estimates: Chetty (2012), Chetty, Guren, Manoli, and Weber (2012).

Beyond Mirrlees

Mirrlees himself in his 1971 paper explicitly write the main assumption about his analysis that have to be relaxed in order to bring more applied insights of his theoretical work:

> Mirrlees setup revealed to be possibly extended to deal with these issues.

This has become the research agenda of public finance economists.

In the introduction of Mirrlees (1971, pp. 175-176), he lists seven assumptions that underlie his analysis:

1. Intertemporal problems are ignored.
2. Differences in tastes ... are ignored.
3. Individuals are supposed to determine the quantity and kind of labour they provide by rational calculation ... and social welfare is supposed to be a function of individual utility levels.
4. Migration is supposed to be impossible.
5. The State is supposed to have perfect information about the individuals in the economy (reported income).
6. Various formal simplifications are made: ... one kind of labour; ... one consumer good; ... welfare is separable.
7. The costs of administering the optimum tax schedule are assumed to be negligible.

Extensive and intensive labor supply decisions

- Extensive labor supply responses were thought to be important in practice.

  Saez (2002), Choné and Laroque (2011), and Jacquet, Lehmann, and Van der Linden (2013).

  Kleven (2024): Based on event studies comparing single women with and without children, or comparing single mothers with different numbers of children, Kleven shows that the only EITC reform associated with clear employment increases is the expansion enacted in 1993. The employment increases in the mid-late nineties are very large, but they are influenced by the confounding effects of welfare reform and a booming macroeconomy.

Extensive and intensive labor supply decisions

- New perspective on the issue: two-brackets reform needed to eliminate Pareto inefficiencies.

  Bierbrauer, Boyer and Hansen (2023). IPP Policy Note (2024)

Bierbrauer, Boyer and Hansen (2023)

- Starting point:
  - Take an income tax-transfer system in place.
  - Is there a tax reform that makes every citizen better off?
  - If yes: Which tax reform does the job?
- We provide
  - empirically applicable Pareto conditions,
  - a test for whether any specific reform is Pareto-improving,
  - a measure of the size of inefficiencies,
  - a tool to identify the "best" Pareto-improving reform.
- We apply these tools to study the 1975 EITC introduction.

What we do

- **Generic formal framework:**
  - Static utility-maximizing choice of earnings.
  - Budget set defined by some non-linear tax schedule.
  - Tax reforms vary the marginal tax in m income brackets (flexibly located).
- **Perturbation approach:** Identify small Pareto-improving reforms.
- **Necessary and sufficient conditions** for Pareto efficiency.
- **Test function** for historical tax reforms.
- **Application** to 1975 EITC introduction in the US.

What we find

- **Two is more than one:**

  If the tax system cannot be Pareto-improved by a one-bracket reform, then there can still be Pareto-improving two-bracket reforms.

- **Two is enough:**

  If the tax system cannot be Pareto-improved by one-bracket or two-bracket reforms, then there is no Pareto-improving reform at all.

- **Express results** using **revenue function** $y \mapsto \mathcal{R}(y)$: revenue gain from a small one-bracket reform at income level $y > 0$.

What we find

- **Application:**
  - 1974 pre-EITC tax system was Pareto-inefficient.
  - 1975 EITC introduction was not Pareto-improving (but close to).
  - Best reform was a larger version of the EITC reform.

- Several sources of taxation:
  - Commodities and income taxation in the model.

    Atkinson and Stiglitz (1976).
  - Capital and income taxation in the model.
  - Jacquet and Lehmann (2021).
- Dynamic aspect of taxation: intertemporal issues, human capital accumulation (Stantcheva, 2017).

You will see this in Part II next week.

Migration decisions

- A lot of talk about migration (specially of the very rich).

  Mirrlees setup can be extended to deal with this issue.

  Tax competition literature developed for capital taxes not for income taxes (see Keen and Konrad, 2013. Handbook of Public Economics).

  Theory: Lehmann, Simula and Trannoy (2014), Bierbrauer, Brett and Weymark (2013), Morelli, Yang and Ye (2012).

  Empirics: recent survey Kleven, Landais, Munoz and Stantcheva (2020).

Frictions on the labor market

- Optimal nonlinear income taxation when there is adverse selection in the labor market.

  Stantcheva (2014): Unlike in standard taxation models, firms do not know workers' abilities and competitively screen them through nonlinear compensation contracts, unobservable to the government, in a Miyazaki-Wilson-Spence equilibrium.

- Income taxation and minimum wage

  Lee and Saez (2012), Cahuc and Laroque (2014), Gerritsen (2017).

Multi-dimensionality of individuals' characteristics

- Individuals different in several observable aspects that correlate with ability: gender, race, age, disability, family structure, number of kinds, height, ...

  Some of these characteristics can be observed by Government, some are not.

1. Does the Government want to condition taxation on them if observable?
2. How does multidimensional heterogeneity change the optimal tax schemes derived?

Multi-dimensionality of individuals’ characteristics: Observable/ can be conditioned on

Tagging: We have assumed that $T(z)$ depends only on earnings z.

Government can observe some of these characteristics X and condition taxation on them: $T(z; X)$.

- Theory results:
    1. If characteristic X is immutable then redistribution across the X groups will be complete (until average social marginal welfare weights are equated across X groups).
    2. If characteristic X can be manipulated (behavioral response or cheating) but X correlated with ability then taxes will still depend on both X and z.

Akerlof (1978), Nichols and Zeckhauser (1982), Weinzierl (2011), Mankiw and Weinzierl (2010).

Multi-dimensionality of individuals' characteristics: Unobservable/ cannot be conditioned on

Individuals different in several unobservable aspects: preferences, health, ...

or government does not want to condition on: height, gender, ...

- Jacquet and Lehmann (2021), Rothschild and Scheuer (2015) Choné and Laroque (2010).

- Individuals decide on their job (which sector to work in) Rothschild and Scheuer (2013, 2014).

Multi-dimensionality of individuals' characteristics: Recent progress

- Analytical and Numerical methods: Spiritus, Lehmann, Renes and Zoutman (forthcoming)
- Carlier, Dupuis, Rochet and Thanassoulis (2024)

- Taxation of couples:
- What is the optimal taxation of couples vs. singles? Should secondary earnings be treated differently?

  Golosov and Krasikov (2025), Bierbrauer, Boyer, Peichl and Weishaar (2024).

Social welfare functions (SWF)

So far: Welfarism = social welfare based solely on individual utilities

Most widely used welfarist SWF:

1. Unweighted Utilitarian: $SWF = \int_i u^i$.
2. Rawlsian (also called Maxi-Min): $SWF = maxmin_i u^i$.
3. $SWF = \int_i G(u^i)$ with $G(.) \uparrow$ and concave, e.g., $G(u) = u^{1-\gamma}/(1-\gamma)$ (Utilitarian is $\gamma = 0$, Rawlsian is $\gamma = \infty$).
4. General Pareto weights: $SWF = \int_i \mu_i \cdot u^i$ with $\mu_i \geq 0$ exogenously given.

Social marginal welfare weights

Key sufficient statistics in optimal tax formulas are **Social Marginal Welfare Weights** for each individual:

Social Marginal Welfare Weight on individual i is $g_i = G'(u^i)u^i_c/\lambda$ (λ multiplier of gov't budget constraint) measures $ value for gov't of giving $1 extra to person i

g_i typically depend on tax system (endogenous variable).

Utilitarian case: g_i decreases with z_i due to decreasing marginal utility of consumption.

Rawlsian case: g_i concentrated on most disadvantaged (typically those with $z_i = 0$).

More general welfare functions

- Social planner objective function is problematic.

  Special aggregation of preferences.

  Many dimensions of desirable redistribution.

  Problem with tagging: Horizontal Equity concerns (people with same ability-to-pay should pay the same tax) impose constraints on feasible policies

$\Rightarrow$ not captured by utilitarian framework.

Saez and Stantcheva (2016), Weinzierl (2014,2010), Fleurbaey and Maniquet (2006, 2011).

Political economy of income taxation

- Social planner is an important benchmark for choice of income taxes.

  However, income tax schedules we observe results of political competition in modern democracies.

Huge literature in political economy to show the outcome of political process when tax instruments are restricted

- Normative Approach: Mirrlees, 1971; optimal non-linear income taxation/ mechanism design.
- Political Equilibrium: Roberts (1977), Meltzer-Richard (1981); linear income taxes.

This makes it difficult to answer the question whether political competition yields desirable outcomes.

There is also no answer to the question whether optimal policies can be decentralized as a political equilibrium.

Only recently solved for the Mirrleesian setup: Bierbrauer and Boyer (2016), Brett and Weymark (2017).

Downsian political competition

Bierbrauer and Boyer (2016) show that political economy outcome in "pure" political competition leads to

Theorem 1: Outcomes are ex post-efficient.

Theorem 2: Policies that trade-off equity and efficiency have no chance.

- First welfare theorem for political competition: efficient policies achieved by political process.

  But ex-post efficient: FIRST-BEST

  No distortionary taxes

- Failure of second welfare theorem: no redistributive taxation based on types politically sustainable

  No redistribution based on types i.e. no "$g(\omega)$".

Probabilistic Voting

Bierbrauer and Boyer (2016) show that political economy outcome leads to

- a version of Diamond's *ABC*-formula in which welfare weights are replaced by a "political elasticity".
- Politicians have "market power."

The marginal income tax rate for an individual with type ω is given by

$$T'(y^1(\omega)) := 1 - \tilde{v}_2(\omega, y^1(\omega)) = -\frac{1 - F(\omega)}{f(\omega)} \, (1 - \beta^2(\omega)) \, \tilde{v}_{12}(\omega, y^1(\omega)) \, ,$$

where

$$\beta^2(\omega) := \frac{1}{\lambda} \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \bar{b}^2(\omega, x^1, y^1 \mid x^2, y^2) \, dG^1(x^1) \, dG^2(x^2) \, ,$$

and

$$\bar{b}^2(\omega, x^1, y^1 \mid x^2, y^2) := E\left[b^2(x^1 - h(s, y^1) - \left(x^2 - h(s, y^2) \right) \mid s) \mid s \geq \omega \right]$$

is the measure of voters politician 1 can attract by offering more utility to type ω-individuals, for a given tax schedule y^2 of the opponent, and conditional on the offers of pork being equal to x^1 and x^2.

Bierbrauer, Boyer and Peichl (2021, AER): Politically feasible reforms of non-linear tax systems

Political economy and welfare-maximizing approaches to redistributive taxation:

I. Normative analysis:

Non-linear taxation, workhorse: Mirrlees (1971).

II. Political economy:

Workhorse only for linear income taxation: Roberts (1977), Meltzer and Richard (1981).

No broadly accepted conceptual framework.

Needed for Political Economy analysis of progressivity, top tax rates, earnings subsidies...

Major transformations of tax systems occurring in the last decades hard to reconcile both from normative and political economy perspectives:

1. Significant decline in top income tax rates in many OECD countries (Piketty et al. 2011);
2. Introduction and subsequent increase of earning subsidies (EITC, Prime d'activité);
3. Sharp progressivity in the middle of the income distribution.

Non-technical summary: Towards politically feasible and welfare-improving tax reforms, VoxEU, October 2020.

Overview

An analysis of politically feasible and welfare-improving tax reforms:

- Classical Mirrlees environment with non-linear schedules.
- Consider reforms of a given status quo in tax policy:
  - President Woodrow Wilson Inaugural Address (March 1913): *"We shall deal with our economic system as it is and as it may be modified, not as it might be if we had a clean sheet of paper to write upon; and step by step we shall make it what it should be"*
  - The Trump Plan *"will collapse the current seven tax brackets to three brackets"*

This paper I

Ambition: Propose a conceptual framework for the political economy of reforms of non-linear tax systems.

- Assume that there is some status quo tax policy.
- Characterize tax reforms that are politically feasible (ie. preferred by a majority of voters).
- Characterize reforms that are politically feasible and/or welfare-improving.

This paper II

Part 1: Monotonic reforms

Theorem 1 (Median voter theorem for tax reforms) Given an arbitrary non-linear tax system, a monotonic tax reform is preferred by a majority if and only if it is preferred by the voter with median income.

This paper III

Part 1: Monotonic reforms

Theorem 1 (Median voter theorem for tax reforms) Given an arbitrary non-linear tax system, a monotonic tax reform is preferred by a majority if and only if it is preferred by the voter with median income.

- Monotonic reform: Change in tax burden a monotonic function of income.
- Monotonic reforms can be used to characterize welfare-maximizing tax systems.

This paper IV

Part 2: Detecting politically feasible reforms

> **Theorem 2 (Characterization)** Given a Pareto-efficient tax system, moving towards lower taxes for below median incomes and towards higher taxes for above median incomes is politically feasible.

Possible explanation for high progressivity for middle incomes

Based on Theorem 2,

- Develop a sufficient statistics approach to identify reforms that are in the median voter's interest.
- Upper and lower Pareto bounds for marginal tax rates.

This paper V

Part 3: Empirical application

“History of (Federal) US tax reforms” through the lens of our model (using tax return micro data and NBER TAXSIM microsimulation model)

1. Are reforms monotonic? New stylized fact: yes, by and large
2. Are the reforms in the median voter’s interest? Depends on ETI.
3. Does “the median voter theorem” hold in the data? Yes.
4. Sharp increase of tax rates around the median income: yes.

Total number of possible reforms (#years*#countries):	528	
Total number of reforms:	394	
Number of monotonic reforms:	309	(78 %)
Number of non-monotonic reforms:	85	(22 %)

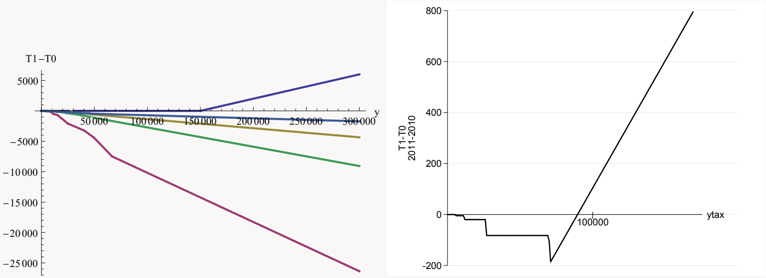
Cuadro: Summary statistics on the tax reforms for a panel of 33 OECD countries (2000-2016).

First year of income taxes:	1916	
Total number of possible reforms until 2016:	100	
Total number of reforms until 2016:	74	
Number of monotonic reforms:	62	(84 %)
Number of non-monotonic reforms:	12	(16 %)

Cuadro: Summary statistics on the history of French tax reforms (1916-2016).

Table 2 is based on the Institut des politiques publiques (IPP) database (accessible on *http://www.ipp.eu/*).

Figura: Important reforms in France



The reforms were implemented in years 2013 (dark blue), 2007 (purple), 2004 (brown), 2003 (green), and 2002 (blue). The figure (right panel) shows the reform implemented in year 2011.

Outline of the class

Introduction

Lecture 2: Tax incidence

Lecture 3: Distortions and welfare losses

Lecture 4-6: Optimal labor income taxation

Outline of the class

Part II: Jean-Baptiste Michau

- Lecture 7: Optimal labor income taxation: The extensive margin
- Lecture 8: Commodity taxation
- Lecture 9: Mixed taxation (commodity & labor income)
- Lecture 10: The taxation of capital
- Lecture 11: Insurance against wage fluctuations
- Lecture 12: Intergenerational taxation