

Public Finance

Pierre Boyer

École polytechnique - CREST

Fall 2026

Outline of the class

Introduction

Lecture 2: Tax incidence

Lecture 3: Distortions and welfare losses

Lecture 4-6: Optimal labor income taxation

Efficiency cost of taxation

- Incidence: effect of policies on **distribution** of economic pie
- Efficiency or deadweight cost: effect of policies on **size** of the pie
- Focus in efficiency analysis is on quantities, not prices

- Government raises taxes for one of two reasons:
 - ① To raise revenue to finance public goods
 - ② To redistribute income
- But to generate \$1 of revenue, welfare of those taxed falls by more than \$1 because the tax distorts behavior
- How to implement policies that minimize these efficiency costs?
 - ▶ Start with positive analysis of how to measure efficiency cost of a given tax system

Marshallian Surplus: Assumptions

- Simplest analysis of efficiency costs: Marshallian surplus
- Two assumptions:
 - ① Quasilinear utility: no income effects, money metric
 - ② Competitive production

Partial Equilibrium Model: Setup

- Two goods: x and y
- Consumer has wealth Z , utility $u(x) + y$, and solves

$$\max_{x,y} u(x) + y \text{ s.t. } (p + \tau)x + y = Z$$

- Firms use $c(S)$ units of the numéraire y to produce S units of x
- Marginal cost of production is increasing and convex:

$$c'(S) > 0 \text{ and } c''(S) \geq 0$$

- Firm's profit at pretax price p and level of supply S is

$$pS - c(S)$$

Model: Equilibrium

- With perfect optimization, supply function for x is implicitly defined by the marginal condition

$$p = c'(S(p))$$

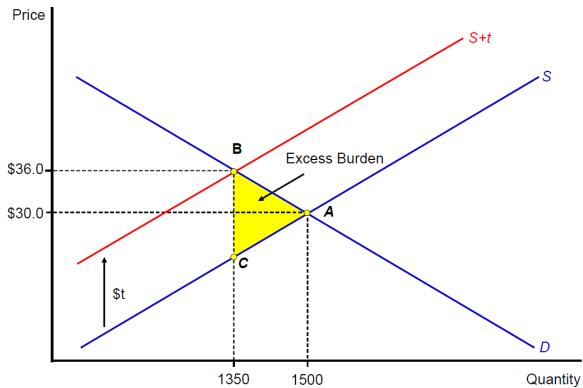
- Let $\eta_S = p \frac{S'}{S}$ denote the price elasticity of supply
- Let Q denote equilibrium quantity sold of good x
- Q satisfies:

$$Q(\tau) = D(p) = S(p + \tau)$$

Let $\eta_D = p \frac{D'}{D}$ denote the price elasticity of demand

- Consider effect of introducing a small tax $d\tau > 0$ on Q and surplus

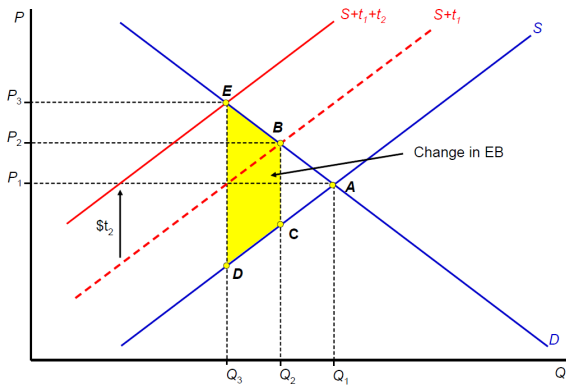
Excess Burden of Taxation



Efficiency Cost: Qualitative Properties

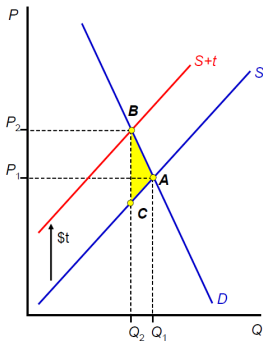
- ① Excess burden increases with square of tax rate
- ② Excess burden increases with elasticities

EB Increases with Square of Tax Rate

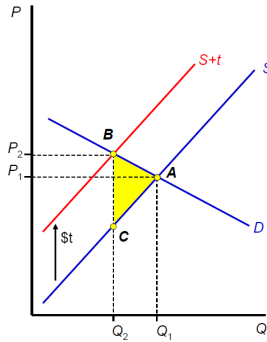


Comparative Statics

(a) Inelastic Demand



(b) Elastic Demand



Tax Policy Implications

- With many goods, the most efficient way to raise tax revenue is:
 - ① Tax inelastic goods more (e.g. medical drugs, food)
 - ② Spread taxes across all goods to keep tax rates relatively low on all goods (broad tax base)
- These are two countervailing forces; balancing them requires quantitative measurement of excess burden

Measuring Excess Burden: Marshallian Surplus

- How to measure excess burden? Three empirically implementable methods:
 - ① In terms of supply and demand elasticities
 - ② In terms of total change in equilibrium quantity caused by tax
 - ③ In terms of change in government revenue

Method 1: Supply and Demand Elasticities

Suppose there is no tax on the good and introduce a small tax $d\tau > 0$ on Q . The excess burden is the size of the triangle:

$$\begin{aligned} EB &= -\frac{1}{2}dQd\tau \\ EB &= -\frac{1}{2}S'(p)dpd\tau = -\frac{1}{2}\underbrace{\frac{pS'}{S}}_{\eta_S} \frac{S}{p} \frac{\eta_D}{\eta_S - \eta_D} d\tau^2 \end{aligned}$$

since $Q = S$, $dQ = S'(p)dp$ and $dp = (\frac{\eta_D}{\eta_S - \eta_D})d\tau$. Therefore,

$$EB = -\frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} pQ * \left(\frac{d\tau}{p} \right)^2$$

- Tax revenue $R = Qd\tau$
- Useful expression is deadweight burden per dollar of tax revenue:

$$\frac{EB}{R} = -\frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} \frac{d\tau}{p}$$

Method 2: Distortions in Equilibrium Quantity

- Define $\eta_Q = -\frac{dQ}{d\tau} \frac{p_0}{Q}$, p_0 initial price,

where η_Q captures the effect of a 1 % increase in price via a tax change on equilibrium quantity, taking into account the endogenous price change (can be identified in the data).

Then,

$$\begin{aligned} EB &= -\frac{1}{2} \frac{dQ}{d\tau} d\tau d\tau \\ &= -\frac{1}{2} \frac{dQ}{d\tau} \left(\frac{p}{Q} \right) \left(\frac{Q}{p} \right) d\tau d\tau \\ &= \frac{1}{2} \eta_Q p Q \left(\frac{d\tau}{p} \right)^2 \end{aligned}$$

Reduced form effect of taxes on quantities (no need to have supply and demand elasticities).

Marginal Excess Burden of Tax Increase

- First, excess burden of a tax τ can be written as

$$EB(\tau) = -\frac{1}{2} \frac{dQ}{d\tau} \tau^2$$

- Now consider EB from raising tax by $\Delta\tau$ given pre-existing tax τ :

$EB(\Delta\tau) = EB(\tau + \Delta\tau) - EB(\tau)$, assuming $\frac{dQ}{d\tau}$ constant (demand curve locally linear)

$$\begin{aligned} EB(\Delta\tau) &= -\frac{1}{2} \frac{dQ}{d\tau} [(\tau + \Delta\tau)^2 - \tau^2] \\ &= -\frac{1}{2} \frac{dQ}{d\tau} \cdot [2\tau \cdot \Delta\tau + (\Delta\tau)^2] \\ &= -\tau \frac{dQ}{d\tau} \Delta\tau - \frac{1}{2} \frac{dQ}{d\tau} (\Delta\tau)^2 \end{aligned}$$

Marginal Excess Burden of Tax Increase

So

$$EB(\Delta\tau) = -\tau \frac{dQ}{d\tau} \Delta\tau - \frac{1}{2} \frac{dQ}{d\tau} (\Delta\tau)^2$$

- First term is first-order in $\Delta\tau$; second term is second-order $((\Delta\tau)^2)$.
- This is why taxing markets with pre-existing taxes generates larger marginal EB.

Method 3: Leakage in government revenue

- To first order, marginal excess burden of raising τ is:

$$\frac{\partial EB}{\partial \tau} = -\tau \frac{dQ}{d\tau}$$

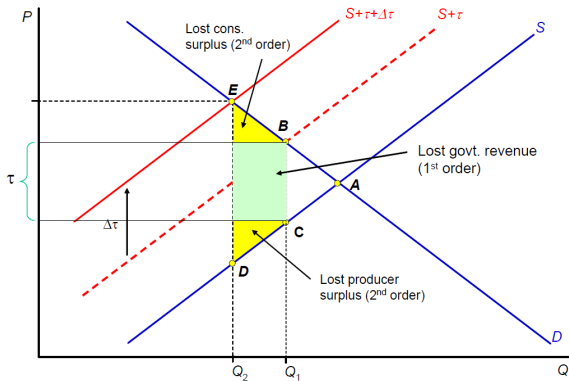
- Observe that tax revenue $R(\tau) = Q\tau$
 - ▶ Mechanical revenue gain: $\frac{\partial R}{\partial \tau}|_Q = Q$
 - ▶ Actual revenue gain: $\frac{\partial R}{\partial \tau} = Q + \tau \frac{dQ}{d\tau}$
- MEB is the difference between mechanical and actual revenue gain:

$$\frac{\partial R}{\partial \tau}|_Q - \frac{dR}{d\tau} = Q - [Q + \tau \frac{dQ}{d\tau}] = -\tau \frac{dQ}{d\tau} = \frac{\partial EB}{\partial \tau}$$

First vs. Second-Order Approximations

- Why does leakage in govt. revenue only capture first-order term?
 - ▶ Govt revenue loss: rectangle in Harberger trapezoid, proportional to $\Delta\tau$.
 - ▶ Consumer and producer surplus loss: triangles in trapezoid (proportional to $\Delta\tau^2$).
- Method 3 is accurate for measuring marginal excess burden given pre-existing taxes but not introduction of new taxes.

Excess Burden of a Tax Increase: Harberger Trapezoid



General Model with Income Effects

- Drop quasilinearity assumption and consider an individual with utility

$$u(c_1, \dots, c_N) = u(c)$$

- Individual's problem:

$$\max_c u(c) \text{ s.t. } q \cdot c \leq Z$$

where $q = p + \tau$ denotes vector of tax-inclusive prices and Z is wealth.

Labor can be viewed as commodity with price w and consumed in negative quantity.

Demand Functions and Indirect Utility

- Let λ denote multiplier on budget constraint
- First order condition in c_i , for all i :

$$u_{c_i} = \lambda q_i$$

- These conditions implicitly define:
 - $c_i(q, Z)$: the Marshallian (“uncompensated”) demand function
 - $v(q, Z)$: the indirect utility function

Measuring Deadweight Loss with Income Effects

- Question: how much utility is lost because of tax beyond revenue transferred to government?
- Marshallian surplus does not answer this question with income effects
 - ▶ Problem: not derived from utility function or a welfare measure
 - ▶ Creates various problems such as “path dependence” with taxes on multiple goods

$$\Delta CS(\tau^0 \rightarrow \tilde{\tau}) + \Delta CS(\tilde{\tau} \rightarrow \tau^1) \neq \Delta CS(\tau^0 \rightarrow \tau^1)$$

- Need units to measure “utility loss”
 - ▶ Introduce expenditure function to translate the utility loss into dollars (money metric).

Expenditure Function

- Fix utility at U and prices at q : Find bundle that minimizes cost to reach U for q ,

$$e(q, U) = \min_c q \cdot c \text{ s.t. } u(c) \geq U$$

Let μ denote multiplier on utility constraint, first order conditions given by:

$$q_i = \mu u_{c_i}$$

- These generate Hicksian (or compensated) demand functions:

$$c_i = h_i(q, u)$$

- Define individual's loss from tax increase as $e(q^1, u) - e(q^0, u)$
- Single-valued function $\rightarrow$ coherent measure of welfare cost, no path dependence.

Compensating and Equivalent Variation

- But where should u be measured?
- Consider a price change from q^0 to q^1 .
- Utility at initial price q^0 :

$$u^0 = v(q^0, Z)$$

- Utility at new price q^1 :

$$u^1 = v(q^1, Z)$$

- Two concepts: compensating (CV) and equivalent variation (EV)
use u^0 and u^1 as reference utility levels.

Compensating Variation

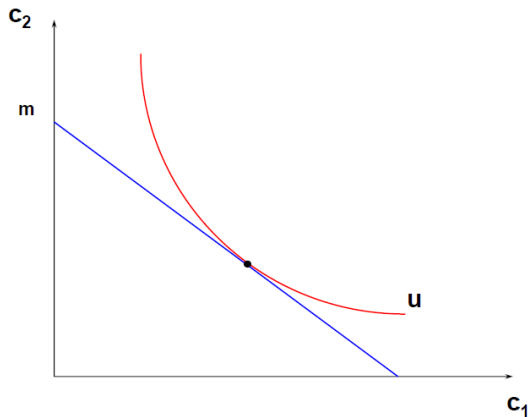
- Measures utility at initial price level (u^0).
- Amount agent must be compensated in order to be indifferent about tax increase

$$CV = e(q^1, u^0) - e(q^0, u^0) = e(q^1, u^0) - Z.$$

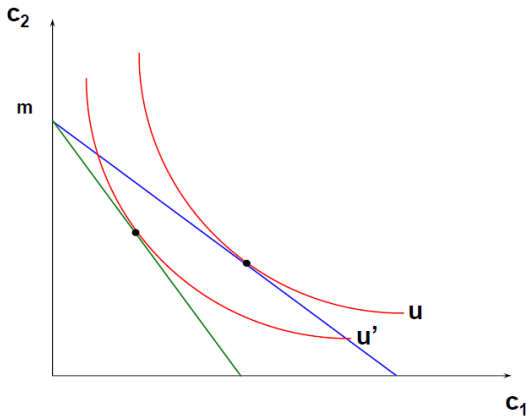
- How much compensation is needed to reach original utility level at *new* prices?
- CV is amount of ex-post cost that must be covered by government to yield same *ex-ante* utility:

$$e(q^0, u^0) = e(q^1, u^0) - CV.$$

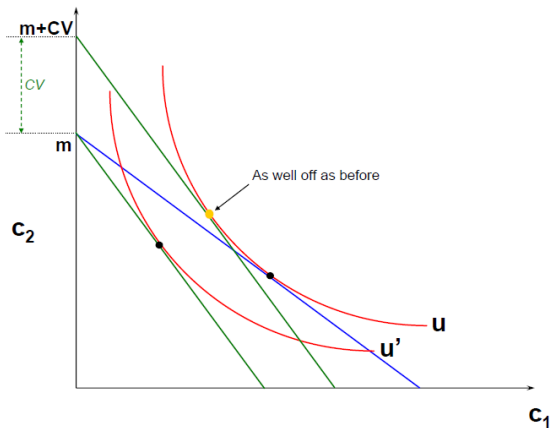
Compensating Variation



Compensating Variation: increase price of good 1



Compensating Variation



Equivalent Variation

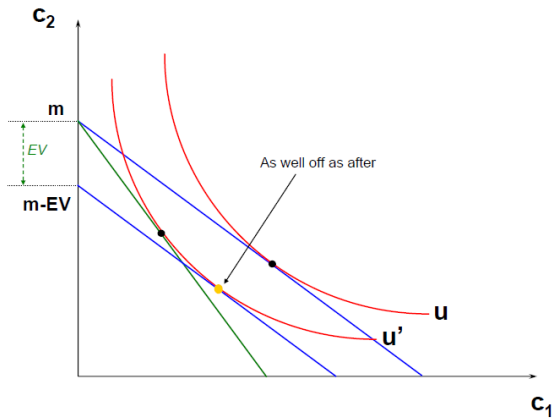
- Measures utility at new price level.
- Lump sum amount agent willing to pay to avoid tax (at pre-tax prices)

$$EV = e(q^1, u^1) - e(q^0, u^1) = Z - e(q^0, u^1).$$

- EV is amount extra that can be taken from agent to leave him with same *ex-post* utility:

$$e(q^0, u^1) + EV = e(q^1, u^1).$$

Equivalent Variation



Efficiency Cost with Income Effects

- Goal: derive empirically implementable formula analogous to Marshallian EB formula in general model with income effects.
- Literature typically assumes either
 - ① Fixed producer prices and income effects;
 - ② Endogenous producer prices and quasilinear utility.
- With both endogenous prices and income effects, efficiency cost depends on how profits are returned to consumers.
- Formulas are very messy and fragile (Auerbach, 1985).

Efficiency Cost Formulas with Income Effects

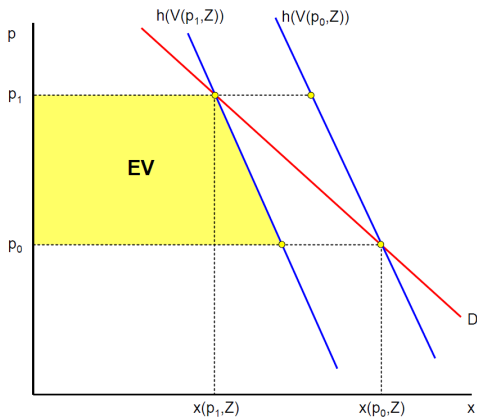
- Derive empirically implementable formulas using Hicksian demand (EV and CV).
- Assume p is fixed $\rightarrow$ flat supply, constant returns to scale.
- The envelope thm implies that $e_{q_i}(q, u) = h_i$, and so:

$$e(q^1, u) - e(q^0, u) = \int_{q^0}^{q^1} h(q, u) dq.$$

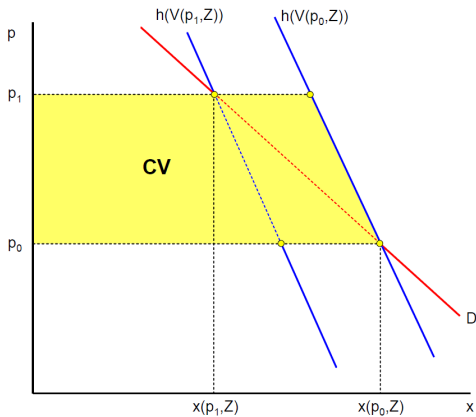
- If only one price is changing, this is the area under the Hicksian demand curve for that good.
- Note that optimization implies that

$$h(q, v(q, Z)) = c(q, Z)$$

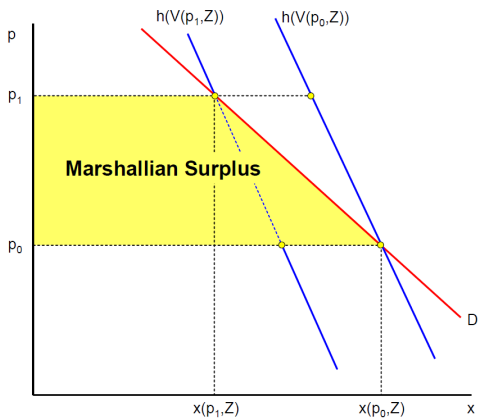
Compensating vs. Equivalent Variation



Compensating vs. Equivalent Variation



Marshallian Surplus



EV, CV, and Marshallian Surplus

- With one price change:

$$EV < \text{Marshallian Surplus} < CV$$

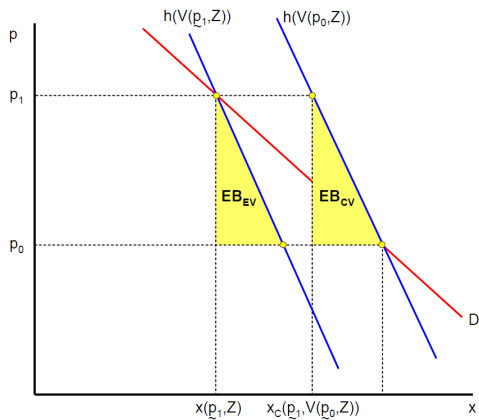
- But this is not true in general with multiple price changes because Marshallian Surplus is ill-defined

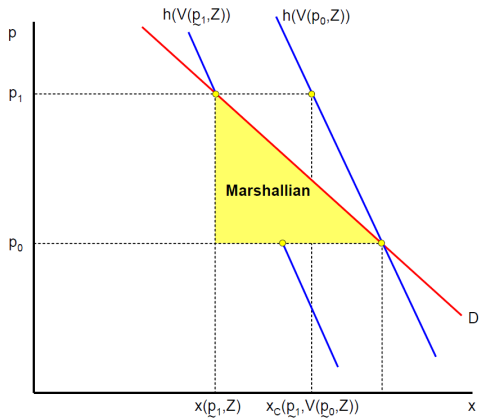
Excess Burden

- Deadweight burden: change in consumer surplus less tax paid
- What is lost in excess of taxes paid?
- Two measures, corresponding to EV and CV :

$$EB(u^1) = EV - (q^1 - q^0)h(q^1, u^1) \text{ [Mohring 1971]}$$

$$EB(u^0) = CV - (q^1 - q^0)h(q^1, u^0) \text{ [Diamond and McFadden 1974]}$$





Excess Burden

- In general, CV and EV measures of EB will differ
- Marshallian measure overstates excess burden because it includes income effects
 - Income effects are not a distortion in transactions
 - Buying less of a good due to having less income is not an efficiency loss
- $CV = EV = \text{Marshallian DWL}$ only with quasilinear utility (Chipman and Moore 1980)

Implementable Excess Burden Formula

- Consider increase in tax τ on good 1 to $\tau + \Delta\tau$
- No other taxes in the system
- Recall the expression for EB :

$$EB(\tau) = [e(p + \tau, U) - e(p, U)] - \tau h_1(p + \tau, U)$$

- Second-order Taylor expansion:

$$\begin{aligned} MEB &= EB(\tau + \Delta\tau) - EB(\tau) \\ &\simeq \frac{dEB}{d\tau} \Delta\tau + \frac{1}{2} (\Delta\tau)^2 \frac{d^2EB}{d\tau^2} \end{aligned}$$

Harberger Trapezoid Formula

$$\begin{aligned}\frac{dEB}{d\tau} &= h_1(p + \tau, U) - \tau \frac{dh_1}{d\tau} - h_1(p + \tau, U) \\ &= -\tau \frac{dh_1}{d\tau} \\ \frac{d^2EB}{d\tau^2} &= -\frac{dh_1}{d\tau} - \tau \frac{d^2h_1}{d\tau^2}\end{aligned}$$

- Standard practice in literature: assume $\frac{d^2h_1}{d\tau^2} = 0$ (locally linear Hicksian); not necessarily well justified b/c it does not vanish as $\Delta\tau \rightarrow 0$

$$\Rightarrow MEB = -\tau \Delta\tau \frac{dh_1}{d\tau} - \frac{1}{2} \frac{dh_1}{d\tau} (\Delta\tau)^2$$

- Formula equals area of “Harberger trapezoid” using Hicksian demands

Harberger Formula

- Without pre-existing tax, obtain “standard” Harberger formula:

$$EB = -\frac{1}{2} \frac{dh_1}{d\tau} (\Delta\tau)^2$$

- General lesson: use compensated (substitution) elasticities to compute EB , not uncompensated elasticities.
- To implement empirically, estimate Marshallian price elasticity and income elasticity. Then apply Slutsky equation:

$$\underbrace{\frac{\partial h_i}{\partial q_j}}_{\text{Hicksian Slope}} = \underbrace{\frac{\partial c_i}{\partial q_j}}_{\text{Marshallian Slope}} + \underbrace{c_j \frac{\partial c_i}{\partial Z}}_{\text{Income Effect}}$$

Excess Burden with Taxes on Multiple Goods

- Previous formulas apply to case with tax on one good
- With multiple goods and fixed prices, excess burden of introducing a new tax τ_k

$$EB = -\frac{1}{2}\tau_k^2 \frac{dh_k}{d\tau_k} - \sum_{i \neq k} \tau_i \tau_k \frac{dh_i}{d\tau_k}$$

- Second-order effect in own market, first-order effect from other markets with pre-existing taxes
- Complementarity between goods important for excess burden calculations
- Ex: with an income tax, minimize total DWL tax by taxing goods complementary to leisure (Corlett and Hague 1953)

Efficiency Cost: Applications

- ① **[Income Taxation]** Feldstein (1995,1999); Chetty (2009); Gorodnichenko, Martinez-Vazquez, and Peter (2009)
- ② **[Housing Subsidy]** Poterba (1992)
- ③ **[Diesel Fuel Taxation]** Marion and Muehlegger (2008)

Welfare Analysis in Behavioral Models

- Formulas derived thus far rely critically on full optimization by agents in private sector
- How to calculate efficiency costs when agents do not optimize perfectly?
- Relates to broader field of behavioral welfare economics
- Two papers if you are interested:
 - ① Conceptual Issues: Bernheim and Rangel (2009)
 - ② Applied Welfare Analysis: Chetty, Looney, Kroft (2009)

Behavioral Welfare Economics

- Abstractly, effect of policies on welfare are calculated in two steps
 - ① Effect of policy on behavior
 - ② Effect of change in behavior on utility
- Challenge: identifying (2) when agents do not optimize perfectly
 - ▶ How to measure objective function without tools of revealed preference?
 - ▶ Danger of paternalism

Behavioral Welfare Economics: Two Approaches

- Approach #1: Build a positive model of deviations from rationality
 - ▶ Ex: hyperbolic discounting, bounded rationality, reference dependence
 - ▶ Then calculate optimal policy within such models
- Approach #2: Choice-theoretic welfare analysis (Bernheim and Rangel 2009)
 - ▶ Do not specify a positive model to rationalize behavior
 - ▶ Instead map directly from observed choices to statements about welfare
 - ▶ Analogous to “sufficient statistic” approach

Behavioral Welfare Economics: Two Approaches

- Consider three different medicare plans with different copays:
 L, M, H and corresponding variation in premiums
- We have data from two environments:
 - ① On red paper, $H > M > L$
 - ② On blue paper, $M > H > L$

Behavioral Welfare Economics: Two Approaches

- Approach 1: build a model of why color affects choice and use it to predict which choice reveals “true” experienced utility
- Approach 2: Yields bounds on optimal policy
 - L cannot be optimal given available data irrespective of positive model
 - Optimal copay bounded between M and H
- Key insight: no theory of choice needed to make statements about welfare
 - Do not need to understand why color affects choice

Directions for Further Work on Behavioral Welfare Analysis

- ❶ Normative analysis of tax policy
 - ▶ Value of tax simplification
- ❷ Use similar approach to welfare analysis in other contexts
 - ▶ Design consumer protection laws and financial regulation in a less paternalistic manner by studying behavior in domains where incentives are clear

Recent reappraisal: Saez and Zucman (2026) A.4

- Provide information on the current distribution of income and tax payments
- Key to quantify income inequality and the direct effects of taxes
- They call it *distributional current-tax analysis*

Background: Conceptual setup for Distributional National Accounts (DINA), Piketty, Saez, Zucman (2018).

Illustration: Two-factor model

Production:

- Aggregate production function $Y = F(K, L)$
- Perfect competition
- w = economy-wide pre-tax wage rate, r = pre-tax rate of return on capital
- Profits maximization: $w = F_L$ and $r = F_K$
- Assume CRS: zero profits $F(K, L) = rK + wL$
- Denote by σ the elasticity of substitution between K and L and by $\alpha = \frac{rK}{Y}$ the share of capital income in the economy.

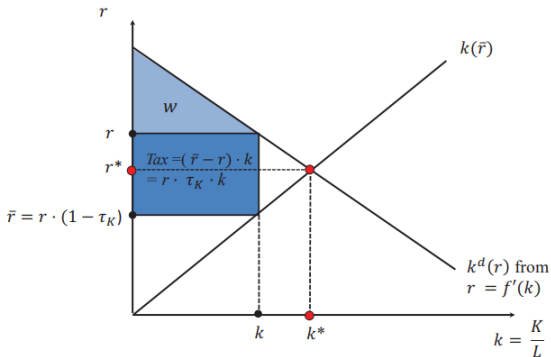
Supply side:

- Assume L is fixed
- Capital income taxed at τ_k
- Capital depends on the net-of-tax return $\bar{r} = r(1 - \tau_K)$
- We can express everything in terms of capital per unit of labor $k = K/L$. As L is fixed, the supply of capital $k = k(\bar{r})$ depends solely on $\bar{r}$.
- Define $f(k) = F(1, K/L) = F(K, L)/L$ as output per unit of labor
 $F_K = f'(k)$ and $F_L = f(k) - kf'(k)$

The equilibrium conditions of the model are:

- $r = f'(k)$ (demand for capital)
- $w = f(k) - kf'(k) = \int_0^k f'(k)dk - rk$ (demand for labor)
- $k = k(r(1 - \tau_K))$ (supply of capital)

Figure 1: General Equilibrium with Capital Tax



Notes: The figure depicts the effect of a tax on capital income at rate τ_K on the interest rate r , the capital to labor ratio $k = K/L$, and the wage w in a general equilibrium neoclassical model with fixed labor L , CRS production $F(K, L) = L \cdot F(K/L, 1) = L \cdot f(k)$. The equilibrium is characterized by 3 equations: (1) $r = f'(k)$ (rate of return of capital equals its marginal return which generates the demand for capital $k^d(r)$), (2) $k = k(\bar{r})$ (capital supply depends on its net of tax return $\bar{r} = r(1 - \tau_K)$), (3) $w = f(k) - k f'(k) = \int_0^k f'(\kappa) d\kappa - rk$ (the wage w can be read as the area below the demand curve and above the r horizontal line). Without taxes, the equilibrium is (r^*, k^*) . With a tax rate τ_K , the equilibrium shifts to (r, k) . The tax collects the rectangle, $(r - \bar{r})k = \tau_K r k$, it increases r , and reduces $\bar{r}$ and w . The tax reduces the wage and the surplus of capitalists by an excess burden triangle $\simeq (1/2) \cdot r \tau_K \cdot (k^* - k)$ over and above taxes collected. In this economy, pre-tax labor income is wL , pre-tax capital income is rK , and post-tax capital income is $r(1 - \tau_K)K$.

Illustration to criticize distributional tax analysis carried out by government agencies

- Current tax analysis: pre-tax income of workers is w
Pre tax income of capitalists is rk , after tax income $\bar{r}k$
- Conventional analysis: Pre-tax income of workers is $w + (r - r^*)k$: neither actual pre-tax income of workers w , nor counterfactual income if no tax.

Empirical Welfare Analysis

Motivation: What government policies do the most to improve social welfare?

- Should we spend more (or less) on health insurance?
- Should we raise top marginal income tax rates?
- Should we invest more in children? At what age?

Normative Evaluation of Policy Changes

- Discuss how to nest causal effects into a normative welfare framework
- Key idea: for each policy change, want to construct its implied **Marginal Value of Public Funds (MVPF)** :

$$MVPF = \frac{\text{Benefits to Recipients}}{\text{Net Govt Cost}}$$

- Hendren and Sprung-Keyser (2020): MVPF translates “reduced form” policy changes into statements about the social welfare impact of those policy changes

MVPF Theory

- Define social welfare:

$$SWF = \sum_i \mu_i u_i$$

u_i individual i 's utility function

μ_i individual i 's Pareto weight

- Define social marginal utility of income: $\eta_i = \mu_i \lambda_i$ where λ_i is the private marginal utility of income

Impact of Policy Change on Social Welfare

- Consider (small) policy change dp , e.g. change in tax rate, educ. subsidy, etc.
- First-order welfare impact:

$$\frac{dSWF}{dp} = \sum_i \mu_i \frac{du_i}{dp} = \bar{\eta}_p \sum_i WTP_i$$

- $\sum_i WTP_i$ is the sum of willingness to pay for the policy by beneficiaries, out of their own income, in \$
- $\bar{\eta}_p = \sum_i \mu_i \frac{WTP_i}{\sum_i WTP_i}$ is incidence-weighted average social marginal utility of income

Reminder: Envelope Theorem

- Consider a parameterized maximization problem:

$$u(p) = \max_y u(y, p) = u(y^*(p), p)$$

- By the FOC, maximization means that when $y = y^*(p)$:

$$u_y(y, p) = 0$$

- By the chain rule for differentiation:

$$u'(p) = \underbrace{u_y(y^*(p), p)}_{=0} y^{*'}(p) + u_p(y^*(p), p)$$

and the envelope theorem just says this:

$$\frac{du}{dp} = u'(p) = u_p(y^*(p), p)$$

Compare Policies by Normalizing by Cost

- Most policies (dp) are not budget neutral
- Let R denote govt budget and $G = \frac{dR}{dp}$ denote net impact on govt budget
- G includes any fiscal externalities from behavioral responses to the policy

- The Marginal Value of Public Funds (MVPF) of policy p is given by:

$$MVPF_p = \frac{\sum_i WTP_i}{G} = \frac{\text{Willingness To Pay}}{\text{Govt Net Cost}}$$

- \$1 of govt spending on the policy delivers:

\$ $MVPF_p$ benefits to the beneficiaries of the policy

$\bar{\eta}_p MVPF_p$ in social welfare ($= \frac{dSWF}{dp} / \frac{dR}{dp}$)

MVPF and Policies that Increase Welfare

- Take two (non-budget neutral) policies: policy 1 and policy 2
- Consider budget neutral policy, dp : increase spending on policy 1 financed from less spending (greater revenue) from policy 2
- To first order, combined policy increases social welfare ($dSWF/dp > 0$) if only if: $\bar{\eta}_1 MVPF_1 > \bar{\eta}_2 MVPF_2$
- MVPFs characterize price of delivering welfare to the beneficiaries through the policy

Motivates comparing policies with similar distributional incidence ($\bar{\eta}_1 \approx \bar{\eta}_2$)

Laffer effect occurs when $WTP > 0$ and $Net\ Cost < 0 \Rightarrow$
 $MVPF = \infty$

Example MVPF: Tax Rate Change

- Let's compute the MVPF a policy that reduces the marginal income tax rate, τ , by $d\tau$

- Let τ denote the marginal tax rate on earnings y .

- Government revenue is $R = \tau E[y]$

where $E[y]$ is the average revenue subjected to the tax

- So, changing taxes leads to a change in revenue

$$\frac{dR}{d\tau} = E[y] + \tau \frac{dE[y]}{d\tau} = E[y](1 + \epsilon)$$

$\epsilon = \frac{dE[y]}{d\tau} \frac{\tau}{E[y]}$ is the elasticity of tax revenue with respect to the tax rate

- Depends on the causal effect of the tax change on tax revenue

- Now, consider the WTP
- Here's where the envelope theorem is useful
- If you earn \$ 100 and taxes go from 10 % to 9 %, WTP \$1 for the decrease regardless of how you change earnings (to first order)

$$\frac{1}{\lambda_i} \frac{du_i}{d\tau} = y_i$$

E.g., quasilinear $u_i = \max_y y - \tau y - v_i(y)$

- So, average WTP is $E[y]$ and the MVPF is given by

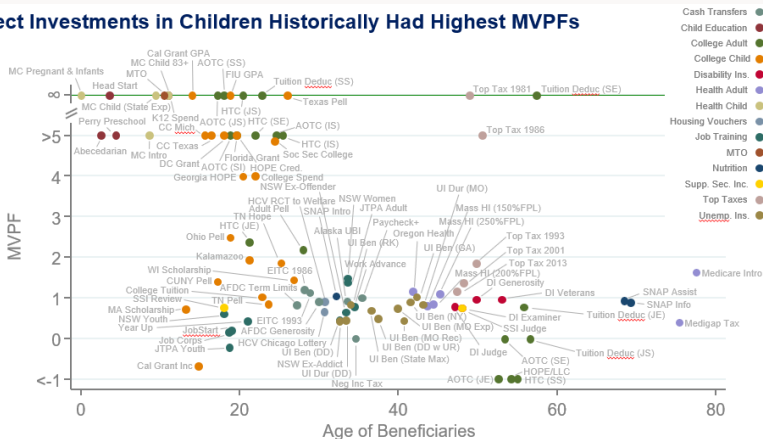
$$MVPF = \frac{E[y]}{E[y](1 + \epsilon)} = \frac{1}{1 + \epsilon}$$

- Key statistic: causal effect of changing tax rates on government revenue: For every \$1 of a tax cut, how much do individuals change their incomes

Empirical Estimates of MVPFs for Various Policies

- Hendren and Sprung-Keyser (2020) construct 133 MVPFs for policies in social insurance, education and job training, taxes and cash transfers, and in-kind transfers.
- New MVPF estimates available at www.policyimpacts.org
- Construct sample from survey and review articles in several policy domains
- Assess robustness to range of assumptions:
 - ▶ Program Parameters (discount rate, effective tax rate , etc.)
 - ▶ Forecasting/Extrapolation of Observed Effects
 - ▶ Validity of Empirical Designs (RCTs/RDs vs. Diff-in-Diff; Peer Reviewed vs. not; etc.)
 - ▶ Publication Bias (Andrews and Kasy, 2019)

Direct Investments in Children Historically Had Highest MVPFs



Link MVPF and MEB

- Marginal excess burden (MEB) corresponds to a distinct policy experiment
- Imagine doing the policy but closing the budget constraint through individual-specific lump-sum taxation (Auerbach and Hines, 2002)
- Rather than scaling up/down different policies to get to budget neutrality as in MVPF framework
- Requires compensated (Hicksian elasticities) not causal effect to calculate MEB

MEB of Tax Rate Change

- Budget constraint is $c \leq \tau y + t$, where t is a lump-sum transfer
- Consider the revenue impact of a tax change that also rebates revenue through changing t

$$\frac{dR}{d\tau^c} = \underbrace{E[y] + \tau \frac{dE[y]}{d\tau}}_{\text{Tax change}} - \underbrace{E[y] - \tau \frac{dE[y]}{dt}}_{\text{Lump-sum rebate}} = \tau \left(\frac{dE[y]}{d\tau} - \frac{dE[y]}{dt} \right)$$

- Normalizing by WTP, $E[y]$, we get

$$MEB = \epsilon^c$$

- ϵ^c denotes the compensated elasticity of tax revenue, subtracting the income effect $\frac{dE[y]}{dt}$

Two issues with the MEB Approach

1. Requires compensated, not causal effects

Income effects are hard to measure (especially if they are not invariant across environments)

2. Individual-specific transfers are not feasible (core idea behind Mirrlees' optimal income tax work: next set of slides)

Bierbrauer, Boyer, Peichl, Weishaar (2026)

- One income source y , one public good g
- Preferences $c + \phi(g, \beta) - K(y, \omega)$; types $\theta = (\beta, \omega) \sim F$: **tastes** and **abilities**
- Budget $c = b + y - T(y)$

Reform package of size s and direction $\tau = (\tau^y, \tau^g)$

- Income tax $T^n(y) = T(y) + s \tau^y h(y)$
- Public good $g^n = g + s \tau^g$
- Transfer $b^n(s) = b + \mathcal{R}(s)$ net revenue is rebated lump sum: the budget balances for every s

For a small reform ($s \rightarrow 0$): who gains? is it politically feasible? does welfare rise?

Revenue: mechanical and behavioral effects

Net revenue after a reform of size s , behavioral responses included:

$$\mathcal{R}(s) = E[T^n(y^*)] - \kappa(g^n) - R^0$$

Marginal revenue effect of a small package

$$\mathcal{R}'(0) = \underbrace{\tau^y(\mathcal{M}^y + \mathcal{B}^y)}_{\text{income tax}} + \underbrace{\tau^g(\mathcal{M}^g + \mathcal{B}^g)}_{\text{public good}}$$

- $\mathcal{M}$: **mechanical** effect at unchanged behavior $\mathcal{M}^y = E[h(y^0)]$,
 $\mathcal{M}^g = -\kappa'(g)$
- $\mathcal{B}$: **behavioral** effect (“leakage” of Method 3), Hendren’s fiscal externality

The $\mathcal{M}$ and $\mathcal{B}$ terms

- **Mechanical**: the change in revenue at unchanged behavior

$$\mathcal{M}^y = E[h(y^0)], \quad \mathcal{M}^g = -\kappa'(g)$$

- **Behavioral**: the marginal tax rate times the earnings response

$$\mathcal{B}^y = E[T'(y^0) y_T^0[h]], \quad \mathcal{B}^g = E[T'(y^0) y_g^0] = 0$$

- From the first-order condition $1 - T'(y) = k_y(y, \omega)$, with iso-elastic effort costs,

$$y_T^0[h] = -\varepsilon \frac{y^0 h'(y^0)}{1 - T'(y^0)} \implies \mathcal{B}^y = -\varepsilon E\left[\frac{T'(y^0)}{1 - T'(y^0)} y^0 h'(y^0)\right]$$

Individual welfare: an envelope result

Lemma 1: Effect on indirect utility of type θ

$$\begin{aligned}\tilde{V}_s(0, \theta) &= \mathcal{R}'(0) - \tau^y h(y^0(\theta)) + \tau^g \text{mwp}^0(\theta) \\ &= \left(\mathcal{M}^y + \mathcal{B}^y - h(y^0(\theta)) \right) \tau^y + \left(\mathcal{M}^g + \mathcal{B}^g + \text{mwp}^0(\theta) \right) \tau^g\end{aligned}$$

- Envelope theorem (Milgrom and Segal, 2002)
- Three mechanical pieces: the **rebate** received, the **tax liability** on status-quo earnings, the **value** of the public good,
 $\text{mwp}^0(\theta) = \phi_g(g, \beta)$

Three criteria for a reform direction

1. Pareto improvement iff $\tilde{V}_s(0, \theta) > 0$ for almost every θ .

2. Political feasibility iff $S^M > O^M$, where

$$S^M = E[\mathbf{1}(\tilde{V}_s(0, \theta) > 0)], \quad O^M = E[\mathbf{1}(\tilde{V}_s(0, \theta) < 0)]$$

3. Social welfare improvement iff $\mathcal{W}_s(0) > 0$, where

$$\mathcal{W}_s(0) = E[g(\theta) \tilde{V}_s(0, \theta)], \quad E[g] = 1$$

Quasi-linearity: g is Hendren's $\bar{\eta}$.

Financing a public good: two packages

Fix the project: one mechanical dollar on g , $\tau^g = 1/\kappa'(g)$, with $\mathcal{B}^g = 0$ its net cost is the mechanical dollar. Two ways to pay for it:

- **Lump sum** ($\tau^y = 0$): nothing else moves; $\mathcal{R}'(0) = -1$ is absorbed by b — everyone pays one dollar
- **Income tax** ($\tau^y = 1/(\mathcal{M}^y + \mathcal{B}^y)$): raises one *net* dollar; $\mathcal{R}'(0) = 0$ — only the direct burden matters

$$\mathcal{W}_s(0) = \frac{E[g \text{ mwp}^0]}{\kappa'(g)} - \begin{cases} 1 & \text{lump sum} \\ \frac{E[g h(y^0)]}{\mathcal{M}^y + \mathcal{B}^y} & \text{income tax} \end{cases}$$

Same project, two packages: **financing matters**.

Relation to the MVPF (I): the same building blocks

- Every instrument has an $\text{MVPF} = \text{WTP} / \text{net cost}$; the marginal cost of public funds of a financing instrument is its MVPF:

$$\text{tax } h : \frac{E[g h(y^0)]}{\mathcal{M}^y + \mathcal{B}^y} \quad \text{project: } \frac{E[g \text{mwp}^0]}{-(\mathcal{M}^g + \mathcal{B}^g)} \quad \text{lump sum: } 1$$

- **Balanced budget.** Each package of the previous slide reads

$$\mathcal{W}_s(0) = \text{MVPF}_{\text{project}} - \text{MVPF}_{\text{financing}}$$

Hendren–Sprung–Keyser’s rule, derived inside the framework.

Earlier example: $\mathcal{M} = E[y]$, $\mathcal{B} = \tau dE[y]/d\tau \Rightarrow$

$$\text{MVPF} = 1/(1 + \epsilon)$$

Relation to the MVPF (II): what the framework adds

- **Explicit budget closure.** The MVPF leaves the financing of a single policy implicit (“relative to another policy”); the MEB closes the budget with individual-specific lump sums that do not exist. Here any residual revenue flows into the **uniform rebate** b , so every direction is a complete policy: revenue-neutral packages leave the rebate unchanged, and lump-sum financing – the rebate as the active instrument – is itself a package ($\text{MVPF} = 1$ under quasi-linearity)
- **Financing as a design margin.** The MVPF ranks given policies; here a package is a **direction** under a constraint, and the choice of *who pays* is part of the policy
- **Incidence at the type level.** The MVPF aggregates at once: $\sum_i \text{WTP}_i$ and one incidence-weighted $\bar{\eta}_p$ per policy. Lemma 1 keeps each type’s gain as a function of **observables** – $h(y^0(\theta))$, $\text{mwp}^0(\theta)$ – against aggregate statistics
- **A positive criterion next to the normative one.**

Takeaways

- Hendren–Sprung–Keyser: “motivates comparing policies with similar distributional incidence ($\bar{\eta}_1 \approx \bar{\eta}_2$).”
- That is exactly where the MVPF stops and this framework continues: who pays the dollar, who gains, and whether the coalition is a majority.

Outline of the class

Introduction

Lecture 2: Tax incidence

Lecture 3: Distortions and welfare losses

Lecture 4-6: Optimal labor income taxation